3.7a Implicit Differentiation

differentiate: term by term wrt x :)

implied relationship between x & y

usually x's & y's are mixed

$y = 1 - x^2$

$\frac{dy}{dx} = -2x$

$x^2 + y = 1$ solve for y → $y = 1 - x^2$ explicit; take der → $\frac{dy}{dx} = -2x$

$2x + x = 0$

$2x + y' = 0$ implicit $y' = -2x$

$y^2 = x$

$(y)^2 = x$ explicit $y = \pm\sqrt{x} = \pm x^{\frac{1}{2}}$ $y' = \pm\frac{1}{2}x^{-\frac{1}{2}} = \pm\frac{1}{2\sqrt{x}}$

$2y \cdot y' = 1$ implicit $y' = \frac{1}{2y}$ or $\frac{dy}{dx} = \frac{1}{2y}$

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Find the slope of the circle at the point (3,-4)

$x^2 + y^2 = 25$

$2x + y' \cdot 2y = 0$

$y' \cdot 2y = -2x$

$y' = \frac{-2x}{2y} = \frac{-x}{y}\Big|_{(3,-4)} = \frac{-3}{-4} = \frac{3}{4}$

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Implicit Differentiation Process

1. Differentiate both sides of the equation with respect to *x*. (term by term)
2. Collect the terms with *dy*/*dx* on one side of the equation
3. Factor out *dy*/*dx*.
4. *Solve for dy/dx.*

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lens, tangents and normal lines

find the tangent and normal lines to the ellipse at the point (-1,2)

$x^2 - xy + y^2 = 7$ product rule?

$2x - (x \cdot y' + y \cdot 1) + 2yy' = 0$

$2x - xy' - y + 2yy' = 0$

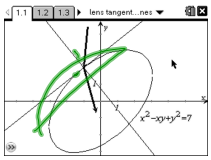
$-xy' + 2yy' = -2x + y$

$y'(-x + 2y) = -2x + y$

$y' = \frac{-2x + y}{-x + 2y}\Big|_{(-1,2)} = \frac{2+2}{1+4} = \frac{4}{5}$

tan line $y = \frac{4}{5}(x+1) + 2$

⊥ line $y = -\frac{5}{4}(x+1) + 2$

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higher order derivatives

Find the second derivative of *y* with respect to *x*

$2x^3 - 3y^2 = 8$

$6x^2 - 6y \cdot y' = 0$

$-6yy' = -6x^2$

$y' = \frac{-6x^2}{-6y} = \frac{x^2}{y}$

$y'' = \frac{y \cdot 2x - x^2 \cdot y'}{y^2}$ quotient rule

$y'' = \frac{2xy - x^2\left(\frac{x^2}{y}\right)}{y^2}$

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